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The Present Situation in Secondary Mathematics, with Particular Reference to the New National Reports on the Place of Mathematics in Education[1]

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Introduction. The subject chosen for this discussion presents so many different aspects that it seemed necessary to agree at the outset on certain limitations of its potential scope. After consulting with the officers of the Association of Teachers of Mathematics of New York City, the speaker decided to stress the following items:

1. The national picture in the field of secondary mathematics.
2. A brief review and comparison of the new national reports.
3. A critique of certain current trends.
4. The emerging rôle of mathematics in a comprehensive system of general education.
5. Some implications and suggestions for teachers, in connection with the impending readjustments in the field of secondary mathematics.

I. *The National Picture.* In his appreciative review of Professor D. E. Smith's *Poetry of Mathematics*, Professor Eric T. Bell included a critical analysis of the current mathematical situation in our schools.[2] His comments may seem too severe, but they were made by an outstanding mathematician and still deserve attention, though written five years ago. To quote,

> In the coming tempest only those things will be left standing that have something of demonstrable social importance to stand on. . . .
>
> The harsh attrition has already begun. Are not mathematicians and teachers of mathematics in liberal America today facing the bitterest struggle for their continued existence in the history of our Republic? American mathematics is exactly where, by common social justice, it should be—in harassed retreat, fighting a desperate rear-guard action to ward off annihilation. Until something more substantial than has yet been exhibited, both practical and spiritual, is shown the non-mathematical public as a justification for its continued support of mathematics and mathematicians, both the subject and its cultivators will have only themselves to thank if our immediate successors exterminate both.
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> Taking a realistic view of the facts, anyone but an indurated bigot must admit that *mathematics has not yet made out a compelling case for democratic support. . . . This must be done, and immediately, if mathematics is to survive in America.*

[1] Based on an address delivered before the Association of Teachers of Mathematics of New York City, at the Washington Irving High School, on April 27, 1940.

[2] See "The American Mathematical Monthly," November 1935, pp. 558 ff.

How true to facts is this arraignment?

A few years ago, Professor Seidlin published the findings based on a questionnaire sent to all the State Superintendents of Education in the United States.[3] He reported that nearly twenty states, including New York State, were no longer requiring a single hour of mathematics beyond the eighth school year. The survey showed conclusively that courses in algebra and geometry, as state required courses for graduation from high school, are "losing ground."[4] Among the reasons for this development, the following were given most frequently by the superintendents: (1) lack of clearly defined and convincing objectives; (2) poor teaching; (3) inadequate textbooks.

Numerous other factors have, however, also contributed to this decline of mathematics in our schools. This list should certainly include the following: (1) a general unawareness of the tremendous significance of mathematics in the modern world; (2) a one-sided emphasis on the doctrine of "social utility"; (3) mechanistic and hence ineffective methods of teaching in primary arithmetic, leading recently to such futile attempts at correction as "stepping up" the entire subject; (4) the absence of a clearly formulated and generally accepted philosophy of education; (5) the doctrines of "progressive education," with their emphasis on immediate experience, individual interests and "felt needs," and their disregard of race experience and sequential learning; (6) a policy of incoherent curriculum revision based largely on momentary interests, "social reconstruction," superficial "orientation," to the exclusion of continuity and foundational training in essential lines of work; (7) the problem of mass education, with the resulting attempts at "adaptation" to individual needs and interests, all of which attempts have been unsuccessful because they have ignored basic causes and problems, have rejected standards, and have preferred an inconsequential tinkering with opportunist expedients or surface adjustments; (8) a wrong psychology of learning based on a mechanistic conception of the mind and avoiding real understanding, thus making the ruination of mathematical instruction almost inevitable; (9) a narrowly specific and hence inadequate training of secondary teachers; (10) the uncertain economic outlook, with a resulting aimlessness and lack of enthusiasm among millions of our young people.[5]

As reflected at the recent meeting of the National Council, in St. Louis (1940), the present situation seems to be characterized by features such as the following:

1. A nation-wide battle between brief, specialized courses in "social mathematics," "consumer's mathematics," "vocational mathematics," etc., and a more broadly conceived program of "cultural mathematics."

2. A great confusion of thought as to

[3] See "The National Mathematics Magazine," various issues, from 1934–1936.

[4] Authentic recent enrollment figures in secondary mathematics do not seem to be available. The Report on "Mathematics in General Education" submits on page 10 a table of "Registrations in Mathematics," reproduced from "School Life," Vol. 22, No. 7. It gives "the number of students in secondary schools and the per cent registered in certain mathematics courses for ten representative states in 1928 and 1934." While the total school population in each of these states *increased* very decidedly during this short period (especially in New York State, namely 46 per cent), the per cent of pupils registered in algebra and in geometry, in these states, *decreased* all the way from 4 per cent to 14 per cent in algebra, and from 2 per cent to 10 per cent in geometry. In New York State the study of secondary mathematics was made optional during these years, with the result that in algebra the number of students declined from about 96,000 in 1928 to about 65,000 in 1934, while in geometry the corresponding figures were 64,000 and 43,000 respectively. The Report states correctly that "these data reveal in quantitative terms a situation which many mathematics teachers have sensed but have not fully realized."

[5] For a more detailed discussion of these and similar factors, see "The American Mathematical Monthly," March, 1936, pp. 132–133; also, various monographs in the *Eleventh Yearbook of the National Council.*

the desirable or possible objectives of either of these types of mathematical training, and as to curricular offerings for the various ability and age levels we have in the high schools.

3. A marked tendency to "adjust" *all* curricula to the lowest ability level, to abandon continuity of training in favor of isolated units or projects, to give up standards of excellence for the sake of the "dull-normal" pupil who "must" be promoted irrespective of attainment.

4. An ineffective handling of the arithmetic situation. (Only a genuine readjustment will bring the desired results. Neither a "remedial program" nor a "stepping up" of the subject can be a substitute for a correct *initial* approach.)

5. A large-scale postponement of secondary mathematics to the later years of the high school. (Thus, "of the 400 high schools of California, between 75 and 100 have pushed algebra and geometry one year ahead into grades ten and eleven, or are definitely planning to do so.")

6. As a result, colleges and universities are now obliged to offer beginning courses in algebra. (This will inevitably lead to college "credit" for such courses.)

7. The situation is now dominated by the general educators and the "curriculum experts," who seem inclined to ignore the views of "subject-matter specialists" and the realistic background of tested classroom experience.

II. *The New National Reports.* In view of the trends outlined above, it is not surprising that another curriculum revision wave of great intensity is now sweeping the country. Thus, Dr. H. Van Engen of Iowa State Teachers College reported at the St. Louis meeting that "the people in charge of the curricula in every hamlet and town feel qualified to revise the curriculum in accordance with their personal points of view." As a result, he says, we now have "chaotic conditions." This corroborates Dr. Bruner's findings of 1937. To quote.

> An indication of the widespread activity in curriculum construction can be found in the increasing number of courses of study that are being produced each year. Prior to 1920 fewer than 1,500 courses of study had been published in the United States . . . . By 1937 approximately 30,000 separate subject courses of study and 10,000 general courses had been received by the Curriculum Laboratory of Teachers College.[6]

It appears that only too often the vastly difficult and responsible task of preparing more adequate curricula for the youth of our country is being imposed upon teachers who have neither the time, nor the equipment, nor the experience for such a job. As a result, there is great danger now that we shall have a period of complete educational anarchy. Reviewing this situation, a prominent critic went so far as to say that this whole enterprise is "not only tragic, but silly." However that may be, it is certain that this movement holds great possibilities for both good and for even greater confusion. And it is in the light of all these developments that we must approach an appraisal of the new national reports. Obviously, their chief function will have to be that of a unifying bond. They are to serve both as a corrective and as a stimulus toward a more scientific approach.

The genesis of these two documents has been made known from time to time in various publications. Their actual messages can be appreciated fully only by reading the reports themselves.[7] It re-

[6] See, "Criteria for Evaluating Course-of-Study Materials," by Herbert B. Bruner, "Teachers College Record," November 1937. Dr. Bruner states that of the 39,046 courses rated in the Curriculum Laboratory since 1924, 3,297 courses had to do with the field of mathematics, and 13.3% of these were considered outstanding

[7] 1. "*The Place of Mathematics in Secondary Education*," Final Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, published as the 15th Yearbook of the National Council of Teachers of Mathematics, by The Bureau of Publications, Teachers College, New York, N.Y., 1940.

2. "*Mathematics in General Education*," Report of the Committee on the Function of Mathematics in General Education, for the

mains to consider some of their chief characteristics, and to attempt a brief comparison of their respective programs.

1. *The Joint Commission Report on the Place of Mathematics in Secondary Education.* This Report aims to deal more definitely with issues which the National Report of 1923, in the opinion of its critics, either treated in too fragmentary a manner, or disregarded entirely. It examines the rôle of mathematics in civilization, relates the teaching of mathematics to the general objectives of education, presents a set of general and specific categories for the building of mathematical curricula, offers two curriculum plans for grades 7–12, has suggestions as to the differentiation of curricula, outlines a program for junior colleges, discusses new methods of evaluation, and submits plans for the education of teachers. Each of these significant items is worthy of detailed consideration. We shall have to limit ourselves here to a review of only two of the principles endorsed by the Joint Commission.

(1) *The Principle of Continuity.* Ever since Professor E. H. Moore's famous address of 1902 (see First Yearbook of the National Council of Teachers of Mathematics) the problem of getting away from the compartment system of teaching secondary mathematics has been under discussion. To this day, we have made little progress in that direction. Until we do, it will be impossible to create a really educational program in mathematics. It has been amply demonstrated that it is impossible, in a single year, to "complete" algebra or geometry. Knowledge which is hastily acquired is speedily forgotten. No "transfer" can be expected from such a brief exposure, in any field of work. And it is unjust to accuse a "subject" for failing to produce "results" when we virtually *prevent* it from functioning. The reports[8] of the International Commission show that in all the other leading countries a much longer time span is allotted to secondary mathematics.[8]

In Chapter V, the J C R outlines a continuous mathematical program for grades 7–12. It suggests one way in which the categories under consideration may receive uninterrupted attention over a period of six years. The plan is not visionary. Most of its major features, especially in the case of grades seven and eight, have been tested in numerous schools over an extended period. It merely remains for progressive school systems to study and put into effect the essential provisions of this program.

What right have we to expect such favorable action at a time when scores of new "subjects" are being sponsored by powerful pressure groups, when "planless" schools favor an unrestricted elective system, and when the demand for ever greater speed causes increasing restlessness and an emphasis on "quick" results?

The answer is twofold. It is based, first, on the demonstrable fact of the vast importance of mathematics in the modern world; and secondly, on the general conviction among virtually *all* high school teachers that a longer time span is needed in all the major subject-matter fields, if solid attainments are to be expected.

As to the *first* factor, we can only refer to the impressive literature which now exists concerning the significance of mathematics in this industrial and scientific era.

The *second* factor has recently received very dramatic attention from so authoritative a body as the Educational Policies Commission. In its well-known volume on "The Purposes of Education in American Democracy" (1938), it used as the motto for Chapter III (The Objectives of Education) the following passage from the writings of the great mathematician and philosopher, Professor Alfred N. Whitehead:

Commission on Secondary School Curriculum, Progressive Education Association, D. Appleton-Century Company, New York, N. Y. 1940.

In the following pages, these reports will be referred to as the J C R and the P E A R, respectively.

[8] See the Fourth Yearbook of the National Council of Teachers of Mathematics.

> There is only *one* subject-matter for education, and that is *Life* in all its manifestations. Instead of this single unity, we offer children—Algebra, from which nothing follows; Geometry, from which nothing follows; Science, from which nothing follows; History, from which nothing follows; a couple of Languages, never mastered; and lastly, most dreary of all, Literature, represented by plays of Shakespeare, with philological notes and short analyses of plot and character to be in substance committed to memory. Can such a list be said to represent Life, as it is known in the midst of the living of it? The best that can be said of it is, that it is a rapid table of contents which a Deity might run over in his mind while he was thinking of creating a world, and had not yet determined how to put it together.[9]

It is significant that at their meeting in Syracuse, last December (1939), the science teachers of New York State discussed for three days the organization of a *twelve-year curriculum in science*, extending from the kindergarten to the university.[10] And the teachers of the social studies are displaying a similar degree of self-consciousness as to their subject. Why should mathematics be given an inferior rôle? Is it really less important in the modern world than either science or the social studies?

Very properly, the J C R regards it as a corollary of the principle of continuity that *we must stop regarding any high school year, except the last, as a* TERMINAL *year in mathematics. Until we absolutely reject the idea of designating "ninth-grade mathematics," or even "eighth-grade mathematics," as the last year of* REQUIRED *mathematics for the academic student of* NORMAL *ability, there can be no real solution of our curriculum problem.* THAT IS THE KEY TO THE WHOLE MATTER.

In spite of all contrary arguments and of dubious educational consequences, mathematical curricula continue to offer *one-year terminal courses in general mathematics*. There may be school conditions which seem to warrant such brief exposure courses. For *slow* or *retarded* pupils they may represent an unavoidable limit of possible attainment. In no case, however, should such courses be regarded as the standard diet for *normal* pupils. It is not an accident that no two of the one-year syllabi in general mathematics which have appeared thus far agree in their offerings. The fact is that it is impossible to build a coherent and significant one-year course in general mathematics, any more than it is possible to construct a satisfactory one-year curriculum in science, or Latin, or English, or any other major educational enterprise. A generation ago, such "get-rich-quick" courses as "Fourteen Weeks in Botany," "Fourteen Weeks in Zoology," and the like, enjoyed a certain degree of popularity. But they have long since ceased to be of interest. Why revive that deplorable mistake in the field of mathematics?

Perhaps it is well to call attention at this point to Professor Dewey's pertinent words of warning:

> Experiences in order to be educative must lead out into an expanding world of subject-matter, a subject-matter of facts or information and of ideas. This condition is satisfied only as the educator views teaching and learning as a *continuous process* of reconstruction of experience. *This condition in turn can be satisfied only as the educator has a long look ahead, and views every present experience as a moving force in influencing what future experiences will be.*[11]

How can a few snapshots of this or that topic provide an adequate vehicle for the *continuous* growth that Professor Dewey rightly emphasizes?

The fact is that all the other leading nations regard mathematics as a major ingredient of any system of general education. They seem to believe that any other attitude, in this technical and scientific

[9] It may be of interest that this statement was included by Professor Whitehead in an address which he delivered in 1912 before the Educational Section of the International Congress of Mathematicians, at Cambridge, England. This address is now Chapter I of Whitehead's "The Aims of Education and Other Essays," Macmillan Company, 1929. Every teacher of mathematics should certainly study this challenging volume.

[10] See "New York State Education," December, 1939, p. 227.

[11] Dewey, John, *Experience and Education*, The Macmillan Company, 1938, p. 111.

age, would have fatal consequences. How can we justify, for pupils of normal ability, any concept of general education that avoids a genuine contact with vitally necessary types of information, skill, and modes of thinking?[12]

(2) *The Principle of Both Objective and Subjective Categories.* The J C R recognizes the necessity of stressing not merely the subject-matter of mathematics, but also the equally important problem of attitudes and of effective *learning*. A mere catalogue of mathematical topics is not sufficient. Until the teachers of mathematics give unremitting attention to the psychology of learning, to questions of motivation, of maturation, of insight, of transfer, real progress will be impossible.

It has been clear for a long time that the negative attitude of many adults, and even of prominent educators, to all phases of mathematical instruction can be traced to their own extremely barren and uninviting contacts with elementary or collegiate mathematics. An inspection of textbooks that were in general use not so many years ago reveals an almost unbelievable degree of pedagogic obtuseness. Quite commonly the pupil was expected to memorize, at the very outset, pages of unrelated definitions, to learn many rules that were never motivated, to complete many hundreds of abstract exercises that were not made meaningful through significant applications, to wrestle with problems that were of no conceivable use to anyone on land or sea, and then to face a final examination which was largely made up of conundrums that only the mathematical élite could attack with any degree of success. "No head for mathematics" was the preposterous verdict pronounced by the pedagogues of that day on a multitude of perfectly capable, otherwise normal youngsters. We are now paying the penalty for a long period of unpardonable errors in the teaching of mathematics.

In order to correct erroneous impressions which are still so widely prevalent, the J C R takes great pains to build mathematics firmly into an enduring framework of any worthy system of general education. It devotes as many as four introductory chapters to a discussion of backgrounds which, in the past, were only too often overlooked or ignored. Both general and psychological considerations are stressed at considerable length. Much space is given to the development of desirable attitudes, of interests and types of appreciation. An attempt is made to explain more clearly than heretofore what we mean when we speak of mathematics as a "mode of thinking." A significant list of guiding principles and of mathematical categories is furnished as a basis of organization of the curriculum. Last, not least, aside from its extensive chapters on the curriculum itself, on evaluation, and on the education of teachers, the J C R supplies, in its Appendix, a new and stimulating analysis of mathematical needs, an informed discussion of the transfer of training, and valuable suggestions concerning the equipment of an up-to-date mathematics classroom.

And so, the J C R offers to the teacher and to the general public a more adequate view of the real significance of mathematics in the modern world.

The preparation of this Report was made possible by a financial grant from the General Education Board which enabled the Joint Commission to hold meetings and to carry on its work. Over a period of years the fourteen members of the Commission, representing the Mathematical Association of America and the National Council of Teachers of Mathematics, gave unsparingly of their time and strength to this difficult enterprise. Their most gratifying reward would be as wide a circulation of the J C R, and as genuine a degree of influence, as was experienced so fittingly by the National Report of 1923.

2. *The Progressive Education Association Report on Mathematics in General*

[12] See again, the Fourth Yearbook of the National Council of Teachers of Mathematics.

Education. This is a very different sort of document from the J C R. It was prepared by a central committee of four members, supplemented by a larger advisory and editorial group. As the Preface explains, this Report constitutes one of a series of publications resulting from the work of the Commission on Secondary Education of the Progressive Education Association, which was established in 1932.

The Report consists of four Parts, comprising a total of thirteen chapters. Part I outlines the underlying philosophy which guided the Committee and discusses the relation of mathematics to the purposes of general education. Part II considers seven "major understandings" growing out of mathematical experience. Part III has as its theme the development and nature of mathematics. Part IV tells us of the importance of understanding the student and presents a novel approach to the problem of evaluating student achievement. Besides, there are two appendices which suggest some of the ways in which the categories treated in Part II may be approached in the classroom.

To the average teacher of secondary mathematics who is accustomed to a precise formulation of objectives, to detailed curriculum outlines in terms of semesters and years, and to conventional, uniform examinations, the P E A R will indeed be very strange diet. And it is more than probable that the general reader will also find it hard sledding to understand and assimilate such broadly oriented, philosophic discussions. Significantly, no fixed curriculum is offered by the P E A R. Instead, it aims to develop so clear a perception of the real purposes and potential contributions of mathematics, in terms of actual life situations, that qualified teachers may be led by this new vision to become creative artists and inspiring guides rather than being mere drill sergeants and followers of prescribed routines.

There will be critics of the P E A R who will maintain that such a program is totally unsuited for our public secondary schools, and that it far transcends their limit of possible achievement. More than that, there is no reason to believe, they will say, on the basis of past performance, that for some time to come even the most outstanding "progressive" schools for which, after all, the P E A R was primarily intended, will have either the wisdom or the courage to translate into action its major recommendations. These critics will assert that, thus far, our private schools and laboratory schools have failed signally, for the most part, to develop a badly needed educational leadership, that they are often less "progressive" than our best public schools, and that not infrequently they have even accentuated the lockstep in education by their almost painful, unwarranted, and evasive regard for traditional college entrance requirements.

The makers of the P E A R would probably admit the justice of such criticisms, as may be inferred from certain passages in the Report itself. But they would urge, by way of reply, that it was the central concern of the Committee, and hence of the P E A R , to change this whole picture. In the preparation of this Report and of its companion volumes, there was a genuine desire to get away from stagnation and "inert ideas," to suggest new horizons and procedures, and hence to make secondary education a more largely autonomous and self-propelling force, of lasting power and beauty, in the lives of our young people. The following paragraphs from the Report may serve as a partial explanation of the attitude of the Committee concerning these problems:

> In formulating the outlines of a program through which mathematical education may advance during the next few years, the Committee had of necessity to be idealistic. To make the proposed program effective and to supply innumerable details of possible content and organization will require experimentation, both extensive and intensive, over a period of years. In the light of such experimentation certain suggestions of this Report will almost certainly need to be modified or even rejected. But with-

out making such recommendations the Committee could hardly hope to outline a forward-looking program, and without such a program mathematics for general education is not likely to become consonant with the needs of the times. . . . This report is addressed primarily to the growing group of *well-trained teachers* who are dissatisfied with the mathematics curriculum in their schools and are seeking a basis for a *fundamental reconstruction* consistent with modern educational theory. . . .

Every effort of this sort, if successful, will in the long run serve to promote the purpose of the Committee—to help teachers of mathematics *better meet the needs of boys and girls.*[13]

Whatever one may think of these opposing positions, it seems certain that for years to come the message of the P E A R will be under debate. Both directly and indirectly its spirit may be expected to influence in marked degree the teaching of secondary mathematics. The failure of the P E A R to furnish a well-defined curriculum will be regarded by some teachers as a fatal weakness, while those who have experienced the crippling influence of too much regimentation will applaud this very defect. In the last analysis, an honest appraisal of this stimulating Report will depend on the educational philosophy of the reader, on his mathematical maturity, and on the degree of his pedagogic self-reliance.[14]

It remains to ask one very important question. Does the P E A R have a central theme or an all-inclusive point of view? The answer is affirmative. It is similar to the famous formulation of the purpose of mathematics found on page 10 of the National Report of 1923, but it involves a much stronger *social* emphasis. The P E A R wishes, above all, to build the teaching of mathematics into a broadly conceived background of educational needs or "basic aspects of living," and to relate it intimately to the *purposes of general education*. One entire chapter is devoted to the latter theme. Its key sentence, printed in capital letters, reads as follows:

THE PURPOSE OF GENERAL EDUCATION IS TO PROVIDE RICH AND SIGNIFICANT EXPERIENCES IN THE MAJOR ASPECTS OF LIVING, SO DIRECTED AS TO PROMOTE THE

[13] See page 14 of the Report.

[14] For a review of current philosophies of education, including that of instrumentalism, and of "progressive" education, see the *Eleventh Yearbook of the National Council of Teachers of Mathematics*, pp. 31 ff.

Dean Herbert E. Hawkes of Columbia University recently reminded us that

Progressive education has no single pattern. It is not standardized. If it were it would not be progressive. Schools that call themselves progressive extend all the way from one where the pupils are asked each morning: "What, if anything, would you like to do today?" to many that are merely alert in attempting to find out by experimentation which of the traditional methods and materials of the conservative schools actually yield results in the intellectual, social and emotional development of youth. (New York Times, September 15, 1940.)

The current literature on progressive education reveals an attempt (1) to identify the movement with everything that has been forward-looking in education during the past two centuries, and (2) to eliminate undesirable or extreme tendencies by the encouragement of honest self-criticism. See, especially, an address by Superintendent Carleton Washburne, President of the Progressive Education Association, on the theme "What is Progressive Education?", published in the *Chicago Schools Journal*, January–February, 1940. Another helpful account is given in *Progressive Education*, May, 1940, under the title "What is Progressive Education Today?", a symposium, by William H. Kilpatrick and others.

The most revealing critique of the movement, by one of its friends, is found in "Progressive Education at the Crossroads," by Professor Boyd H. Bode (New York, 1938).

More severe appraisals, partly in humorous form, are represented by the following articles or papers:

(1) "Lollipops vs. Learning," by Ann L. Crockett, in *The Saturday Evening Post*, March 16, 1940.

(2) "Alice in Cloud-Cuckoo-Land," by I. L. Kandel, *Teachers College Record*, May, 1933.

(3) "An Essentialist's Platform for the Advancement of American Education," by William C. Bagley, in *Educational Administration and Supervision*, April, 1938.

(4) "Progressive Education is Dying? Hail: Saltatory Education," by Superintendent W. Butterfield, in *The Clearing House*, December, 1939.

Of great importance, in connection with the program of the P E A R, will be Dr. Wilford M. Aikin's comprehensive report, now in preparation, on the "eight-year study" which originally gave rise to the P E A R and its companion volumes.

FULLEST POSSIBLE REALIZATION OF PERSONAL POTENTIALITIES, AND THE MOST EFFECTIVE PARTICIPATION IN A DEMOCRATIC SOCIETY.

Part II then attempts to show how mathematics may share in this general program through the development of certain major understandings. *At the very center of this enterprise the Committee places reflective thinking, or problem-solving.* And for the solution of problems, it is stated, the student needs an ever growing appreciation of these concepts: (1) formulation and solution; (2) data; (3) approximation; (4) function; (5) operation; (6) proof; (7) symbolism. A special chapter is devoted to each of these concepts. Each chapter is followed by a helpful bibliography. The discussion is characterized throughout by a scholarly breadth and by a new outlook that is refreshing. Thus, the chapter on symbolism is a most interesting one. By linking this category to the broad sweep of the whole semantic movement, it is shown how this mathematical domain may be robbed of its usual dryness and be made fruitful in unexpected ways.

In conclusion, it may be noted that the seven categories stressed by the P E A R are presented as a sort of mathematical oligarchy, all of them serving the common cause of problem-solving in significant social situations. This is a drastic departure from the point of view made familiar by the National Report of 1923. In that Report, the *function concept* constituted the center of gravity, while in the P E A R its rank does not exceed that of the other central concepts. Perhaps this shift in emphasis will be regarded by a majority of the teachers not only as inherently sound, but also as more in keeping with actual schoolroom practice.

3. *A Comparison of the Two Reports.* Clearly, a new vision is suggested by both the J C R and P E A R. Each desires to relate mathematics to the needs of the pupil and to modern life in the broadest possible sense, though their approaches differ very decidedly. Each believes in continuous mathematical training over a period of years, and each is prepared to endorse the idea that, in the language of Professor Hogben, *the history of mathematics is a mirror of civilization.* In their choice of fundamental categories which should underlie the entire program in mathematics, and in their high regard for general principles, desirable attitudes, interests and appreciations, the two reports are also in close accord. Both look upon the study of mathematics as an indispensable laboratory in clear thinking, and they stress this aspect of mathematical teaching in every possible way. But whereas the J C R presents its principal discussion of the selected categories in the brief space of less than ten pages, the P E A R devotes more than forty per cent of its 412 pages to a consideration of "the major understandings growing out of mathematical experience." An itemized comparison of these sections of the two reports is, of course, out of the question. It may be of interest, however, that the reports present very similar views as to the relative importance of the *function concept.* Thus, the J C R informs us that

> The ambition to make mathematical instruction more broadly significant through emphasis on concepts has led to stressing the *function concept* as a unifying element. Inasmuch as it deals with relationships, it is quite true that few concepts have greater universality or importance. A society, all members of which while in school have been given persistent and effective contact with this concept, should view problems and situations more intelligently than a society which has only a certain number of mathematical specialists. But *the great importance of the function idea should not lead to an over-emphasis upon its significance,* nor should it lead to slighting mathematics which does not come under its scope, for very important and very interesting parts of the subject are unrelated to it.[15]

The P E A R approves of this position, but wishes to create a wider perspective for functional thinking. To quote,

[15] See page 41 of the J C R.

It must be noted that the idea of function permeates modern mathematics, and has been suggested by some authorities as the concept best suited *to unify instruction in this field. Under a narrow quantitative definition of function, complete unification of mathematics is not possible.* However, by employing the term *function* in a broad sense much of the system of elementary mathematical concepts may be embraced. Since the publication of the *Report of the National Committee of 1923* there has been greatly increased emphasis upon the function concept in the teaching of secondary-school mathematics. Teachers and students have become familiar with some of the basic notions, but misconceptions are still prevalent concerning important points. The discussion which follows is not intended to be an introduction to the subject, but rather a commentary upon some aspects of the function concept which are most often overlooked.[16]

The distinction between these two reports can perhaps be symbolized, at least roughly, as follows: If "S" represents "*systematic* mathematical training," while "L" denotes "a series of significant *life situations* suggesting certain mathematical backgrounds," then the J C R may be called an $S \rightarrow L$ report, while the P E A R is an $L \rightarrow S$ report. Hence one might also say that the J C R proceeds *deductively*, while the approach of the P E A R is *inductive*.

In a footnote on page 14, the authors of the P E A R themselves attempt a partial comparison of the two reports with which we are concerned. It is significant enough to merit its inclusion at this point. Its object is to explain why the authors of the P E A R preferred not to submit definite curriculum plans. To quote,

> It is with respect to this point that the task undertaken by this Committee differed from that of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics. In preparing its report on "The Place of Mathematics in Secondary Education," the Joint Commission, after discussing the general aims of education, sought to outline a program of the sort being offered at the moment by some schools in advance of the great majority. Most of its suggestions have been tested to some extent in practice, and *the Joint Commission took a practical rather than experimental point of view.*

The concluding sentence of the foregoing paragraph states the difference between the two reports only in a formal sense. It does not touch on the vital point at issue. Both reports might have been equally "experimental" and yet be very far apart in their basic philosophies of education.

The underlying melody of the P E A R, after all, is unmistakably the one which has become identified with the functionalism and instrumentalism of the Dewey-Kilpatrick group of educators. If one is in general agreement with Dewey's theory of knowledge and his naturalistic doctrine of experience, all is well. But if one is forced, on the basis of classroom experience, to reject a large part of that educational gospel—as essentially unworkable under conditions of mass education such as we face today—one is led to a contrary appraisal.

It would take much space to elucidate this all-important matter. Copious volumes have been written about it.[17] Until our conflicting philosophies are clarified, confusion will continue to prevail in our educational literature.[18]

Now, the "instrumental" orientation of

[16] See page 140 of the P E A R; also, page 70. This whole question deserves careful study and re-examination by all teachers of secondary mathematics. The tremendous importance of *functional thinking* should certainly be stressed in every possible way, as Professor E. R. Hedrick has so often insisted. But perhaps our classroom methods have given only perfunctory and incidental attention to the function concept. It has been treated as an added "topic," and not as an organizing principle or a basic category. The views of teachers on this vexing issue will be clarified by a careful reading of the *Ninth Yearbook of the National Council of Teachers of Mathematics*, which contains Dr. H. R. Hamley's foundational monograph on "Relational and Functional Thinking in Mathematics."

[17] See, for example, Childs, John L., *Education and the Philosophy of Experimentalism*, The Century Company, 1931.

[18] See, especially, Dewey, John, and Childs, John L., "The Underlying Philosophy of Education," Chapter IX of *The Educational Frontier*, D. Appleton-Century Company, 1933.

the P E A R is clearly revealed by key sentences such as the following:

> The development of intelligence in analyzing problem situations, otherwise referred to as reflective thinking, although but a part of the purpose of general education, is so essential a part as to be given a major place in this Report. . . . It is here that teachers of mathematics can make their major unique contribution. . . . *The Committee advocates planning curricular sequences primarily on the basis of concrete problems encountered in meeting educational needs in these four areas, rather than on the basis of logical sequences of the familiar sort, or separate subjects like algebra, plane geometry, solid geometry, and so on, or unifying concepts presented here or elsewhere. . . . A mathematics curriculum may be built by locating and studying concrete problem situations which arise in connection with meeting needs in the basic aspects of living.*[19]

Much of the discussion pertaining to problem-solving in the P E A R is excellent, such as the demand for a more unified approach in mathematics. Moreover, the difficulties involved in working out a practical program along these lines were frankly recognized by the Committee (see pp. 71 ff. of the Report). But one wonders whether the Committee fully realized that the essential features of this problem approach, though never formulated with convincing clearness, have been tried again and again, and *always without success.* What has become of the work of such men as Branford, Mair, and Perry in England? Who remembers the valiant efforts of a *vigorous* group of Herbartians on behalf of a life-situation approach? Why did Dewey's original laboratory school in Chicago come to an early end, in spite of his crusading spirit, his "School and Society," and Katherine Dopp's beautiful, implementing books? Why did the "real problem" movement sponsored for a long time by "School Science and Mathematics" fail to accomplish the desired results? Why do we hear far less today about "Fused Mathematics," "Unified Mathematics," and the like? Why did Ligda's problem approach in algebra, endorsed so enthusiastically by J. W. Young, make no lasting impression? What caused the whole problem-project movement to calm down so soon? And why have life situations proved inadequate even in the teaching of elementary arithmetic?

In this connection it is also helpful to refer briefly to an analogous situation in the field of *science* in our secondary schools. Which domain can boast of a greater potential wealth of "real" problems that should be of immediate interest to adolescent boys and girls,—problems of electric energy and electric appliances, of motors and engines, of radios, automobiles, airplanes, of health and human welfare, not to mention an almost endless array of related social, economic, and industrial aspects of these problems? And what use has been made in the schools of this immense reservoir of significant quantitative backgrounds? Very little, in spite of splendid laboratories, excellent books, and wonderful teaching aids such as projection lanterns and films. To be sure, alert teachers, both of mathematics and of science, have advocated again and again a closer correlation or even an integration of these great fields of learning. And brave attempts in this direction have been made from time to time by a few pioneering enthusiasts. But these endeavors have regularly failed to bring about a modification of our school procedures. Instead, there has developed a veritable flight from problem-solving in our science teaching, if one may judge existing trends from current texts, published reports, and recent curricula. Apparently, science courses are becoming largely informational and descriptive.

Again we are forced to ask, Why? Science teachers tell us that for many pupils scientific problems are too "hard," that very often their pupils do not know the required mathematics, and that *they find it impossible to teach the necessary mathematical techniques incidentally.* On the other hand, the teachers of mathematics usually ignore the whole realm of science for precisely similar reasons. They say that *scientific concepts and principles*

[19] See pp. 52, 53, 72, 73 of the P E A R.

cannot be taught incidentally in the mathematics classroom, within the customary time limits. And so the dilemma continues to exist, to the detriment of both fields.

A long and illuminating review could easily be written concerning these persistent attempts, undertaken by high-minded leaders, to make all school work, including mathematics, more significant and more appealing. The undeniable failure of all these movements must have had weighty causes. Are we to suppose that in all these cases we may conveniently put all the blame on the alleged "lag" of our schools, or on the "incurable" conservatism of teachers and administrators? The only other conclusion would be that there must have been serious inherent weaknesses in the underlying educational programs.

The latter conclusion seems, in fact, inescapable, for an exclusive dependence on life situations and concrete or "real" problems, so it appears, cannot be dissociated from fatal pedagogic difficulties which it has not been found possible to overcome. The essence of these difficulties may be summed up as follows. A real problem in mathematics involves a background of concepts, principles, and skills without which its solution is impossible. The acquisition of this background cannot be forced, but requires a time span which varies with different groups of pupils. Hence only two roads are open to the teacher. *First*, he may dwell at length on a selected life situation, until the pupils have mastered the necessary techniques. This is often a very tiresome, boring procedure, which puts a considerable strain on classroom morale, especially in retarded classes. *Second*, he may pass more quickly from one problem situation to another, and depend on *incidental* practice exercises for the development and maintenance of the requisite mathematical skills. In that case, however, there will either be an insufficient amount of practice, or else there arrives a point—very soon—where the essential building of concepts and skills no longer parallels the problem-solving enterprise. But this means that we are thrown back on the usual separation of "interest and effort," of theory and application, the very situation which the problem-approach was expected to remedy.

Until it is shown, perhaps by a supergenius in education surpassing even Professor Dewey, how this dilemma may be overcome, the average teacher of mathematics will continue to view with justifiable alarm the demand that problem-solving be made the central concern of the curriculum.

In closing this very fragmentary comparison of the two reports, we may summarize our impressions by saying that in reality these documents *supplement* each other. Together, they constitute a challenge to every progressive teacher of mathematics. But in so far as the P E A R offers no curriculum, and ultimately depends on more or less unrelated and uncharted life situations to be made mathematically significant by the genius of teachers and pupils, it represents a vastly more difficult program. In fact, for the average teacher and pupil it must be regarded as futuristic, though extremely helpful and stimulating. The whole doctrine of life situations, while most fascinating in theory, unquestionably involves a cluster of unsolved and very baffling problems. Some of these we shall consider in a later section. Perhaps we shall then be enabled to arrive at an unbiased and objective view concerning the relative merits of the new reports and concerning their probable effect on the future development of mathematics in our secondary schools.

III. *A Critique of Certain Recent Trends.* Of the numerous "trouble zones" in the present situation, only three will be considered. Each has been a cause of great confusion, and a source of much uneasiness among teachers.

1. *The Doctrine of Immediate Interests.* For a discussion of the gradual evolution of this doctrine, and of its great influence on current curriculum procedures, we must

refer to the professional literature on the subject. Closely related to it is the doctrine of "incidental learning." When the curriculum is supposed to "emerge" from day to day, under the stimulus of actual life situations discovered or suggested by the pupils themselves, without a semblance of a sequential plan, the result can only be chaos. A trenchant critique of this educational aberration may be found in recent pronouncements of such authorities as Dewey, Bode, Bagley, Hanna and Kandel.[20]

2. *The Doctrine of Social Utility, and of "Functional" Values.* This is, perhaps, the very center of the present storm area in education. With ever-increasing vehemence the feeling has been gaining ground that secondary education has become, in the language of Professor Whitehead, a domain of "inert ideas." These are ideas, he says, which are "merely received into the mind without being utilized or tested, or thrown into fresh combinations." He tells us, in ringing phrases, that "education with inert ideas is not only useless; it is, above all things, harmful." He asserts, very positively, that "education should be useful, whatever your aim in life." In fact, he defines education as "the acquisition of the art of the utilization of knowledge." But the same author is even more insistent on the retention of real thinking and on thorough, consecutive training. "Theoretical ideas," he says, "should always find important applications within the pupil's curriculum." We should "begin to treat algebra as a serious means of studying the world." He enters a plea for the development of Power, "an intimate sense for the *power* of ideas, for the *beauty* of ideas, and for the *structure* of ideas."

This is a very different program from the narrow utilitarianism of many curriculum theorists. And we must test recent applications of the doctrine of "social utility" and of "functionalism" by the touchstone embodied in Professor Whitehead's criteria.

It now appears that curriculum "experts" are in danger of repeating in the secondary field the mistakes which were made in the elementary field by an extreme insistence on "social utility." *Against that calamity we must be on guard.* Nothing would pay greater dividends just now than a careful study of the way in which extreme tendencies almost wrecked the teaching of arithmetic.[21] To be sure, it is absolutely true that the doctrine of social utility, correctly applied, has served as a powerful stimulus in vitalizing arithmetic. More than ever, we now realize that social backgrounds and life situation problems are *essential* for purposes of motivation and of transfer. On the other hand, arithmetic is, above all, a generalized mode of thinking. And Professor McConnell reminded us at the recent St. Louis meeting that social backgrounds, as such, "do *not* force meaningful learning in arith-

[20] The following volumes or papers are typical of the rising tide of criticism directed against "aimless" or "planless" education:

(1) Dewey, John, *Experience and Education*, The Macmillan Company, 1938.

(2) Dewey, John, *The Way out of Educational Confusion*, Harvard University Press, 1931.

(3) Bode, Boyd H., *Progressive Education at the Crossroads*, New York, 1938.

(4) Bagley, William C., "An Essentialist's Platform for the Advancement of American Education," in *Educational Administration and Supervision*, April, 1938.

(5) Hanna, Paul R., and others,"Opportunities for the Use of Arithmetic in an Activity Program," in the *Tenth Yearbook of the National Council of Teachers of Mathematics*, New York, 1935.

(6) Kandel, I. L., *Conflicting Theories of Education*, The Macmillan Company, 1938.

[21] A very valuable and sound discussion concerning this question of social backgrounds in arithmetic is that of Professor B. R. Buckingham in the *Twenty-Ninth Yearbook of the National Society for the Study of Education* (1930). See Part I, Chapter II of that yearbook (pp. 8–62) on "The Social Value of Arithmetic." The dangers of the "Social utility" movement may be inferred from such studies as that of Bowden, A. O., *Consumer's Uses of Arithmetic*, Teachers College, Columbia University, 1929, and from the report by Dr. Paul R. Hanna and others on "Opportunities for the Use of Arithmetic in an Activity Program," in the *Tenth Yearbook of the National Council of Teachers of Mathematics.*

metic." That is, the concepts, skills, and principles of arithmetic are not inferred automatically from concrete situations. They must be learned *mathematically*, as related elements of a closely knit system of ideas and processes. Similar remarks apply even more forcibly, of course, to the field of secondary and higher mathematics.

If all this is admitted, we can feel rather certain about the defects of the present movement to limit our secondary schools to a narrowly conceived program of "social mathematics." We may admit without reservation the great potential value of life situation backgrounds. We should welcome a greatly increased emphasis on "real" problems. But we should not be blind to the ruinous effect of a *purely utilitarian* orientation. As to the undeniable dangers and weaknesses of this trend, the following considerations seem to warrant special attention:

(1) *Life situations are* NOT *a substitute for thorough mathematical training. They do* NOT *guarantee the learning of mathematical ideas, principles and techniques.*

(2) It has been found impossible, even in the field of arithmetic, to arrange life situations *sequentially* in such a way that mathematical concepts, principles and processes can be built up, with their aid, in the *cumulative* manner which is essential in mathematics.

(3) In secondary mathematics an adequate solution seems even more difficult. A generation ago the Perry movement in England, with its passionate devotion to practical applications and its contempt for "mere theory," failed completely. Likewise, the movement to correlate mathematics and science, emanating from Chicago at about the same time, was not successful. A similar fate overtook all attempts to vocationalize secondary mathematics.

(4) In each of these cases, failure was due to the same basic causes, as follows: (a) the absence of a coherent program; (b) an almost total neglect of real understanding, insight and mastery; (c) a shocking disregard of the principles of learning; (d) a bunching of too many unfamiliar elements in a single learning situation; (e) an inadequate time allowance which often crowded into the short period of a semester or a year an array of items that the average pupil simply could not assimilate.

(5) These perennial mistakes are in evidence, once again, in virtually all recent courses published under the title of "social mathematics," "consumer's mathematics" and the like. In only too many instances, there are no adequate criteria of selection, of arrangement, of degrees of emphasis, and the like, such as are sugested by the J C R. And only too often the course as a whole is pathetically inadequate. That is, such courses have "socialized" mathematics by virtually *omitting the mathematics* they were supposed to elucidate and to apply.

(6) Let it be stated again that we should certainly be grateful for every intelligent attempt to make the study of mathematics more significant and interesting. But so long as the errors mentioned above cannot be eliminated by curriculum builders, authors and teachers, the program known under the name of "social mathematics" can hardly be regarded as the ideal solution of our difficulties. Thus far, it has not aided in the preservation or reconstruction of either algebra or geometry, except in the direction of drastic simplifications and eliminations. In the main, it has consisted in moving the business and social applications of eighth-grade *arithmetic* into the upper high school years. It has avoided the really vital task of building a new foundation for arithmetic. And it has indulged in an almost naïve enthusiasm on behalf of such topics as installment buying, owning and operating a car, budgeting, and the like. These things are all very interesting, and, to a certain extent, necessary. But to regard them as the great panacea, as the "be-all and end-all" of *secondary* mathematics, is a most pathetic development.

Such a plan completely ignores other, equally important, life needs of many groups of pupils. Above all, it does not effect a sufficiently broad understanding of the rôle of mathematics in the modern world.

The doctrine of social utility and of "functional" values is, of course, one aspect of the program endorsed by the P E A R. That program, as was said above, is not an easy one. This fact is recognized very openly and honestly by the makers of the P E A R. Thus, in a discussion of the P E A R at the 1939 meeting of the National Council, Dr. M. L. Hartung closed his remarks with these words:

> To build a curriculum along these lines we must sentence ourselves to *many years of hard labor*. But it is not labor on the rock pile. It is labor upon a more glorious mathematical edifice in which the boys and girls of the next generation may enthusiastically dwell.

3. *The Vocational Aspect of the Situation.* What are we to think, finally, of the constantly recurring attempts to vocationalize secondary mathematics, to make it talk the language of the shop, the industrial plant, the accounting office, and to spread on the pages of elementary textbooks a thin veneer of formulas about automobiles, airplanes, radios, and the like, especially when there is no opportunity whatever for infusing meaning and significance into such backgrounds? In a sense, this trend is a corollary of the socialization movement. It aims to restrict all curricular offerings to the vocational and hence utilitarian interests of the student. As a result, it is opposed to a broadly cultural type of program. The arguments against general orientation and really basic training are along these lines; (a) there is no time for such a program; (b) the average student is not interested in anything but his immediate needs. It has been claimed that if the majority of our secondary pupils could be enrolled at once in vocational or trade schools, the problem of motivation would in large measure be solved.

A candid study of this vocational trend would seem to justify conclusions and observations such as the following:

(1) The reports of the American Youth Commission make it apparent that four million of our young people between the ages of 16 and 24 are now unemployed. And the immediate outlook for prompt occupational employment upon graduation from school is not very bright. (These findings are in harmony with those of the New York State Regents' Inquiry Reports.)[22] The whole situation is a very complicated one. This makes all forms of early vocational training a rather difficult problem.

It is possible, of course, and even probable, that the present world crisis may completely change this picture.

(2) It is not true that vocational courses can, as a rule, dispense with a relatively thorough training in fundamentals. Exactly the reverse is the case. A pupil who is never sure of his basic techniques, who cannot place a decimal point correctly, cannot measure, does not understand percentage, is unfamiliar with geometric concepts and principles, will almost certainly fail in many practical lines of work. Moreover, contrary to the opinion of certain optimistic curriculum "experts," such training cannot be acquired "incidentally" or by a few sporadic lessons.

(3) It has been shown by competent investigators that even at the lower levels of many vocational courses a broad type of basic training is essential. And the more desirable positions in shops, in offices, in laboratories, and in industrial plants, now demand virtually complete high school and even college courses.

We may summarize these considerations by quoting the following passage from a recent authoritative article on "Youth and Vocational Education," by Mr. Oakley Furney, chairman of a committee on vocational training:

> Vocational schools should provide *all-around training* for occupations. Both skills and knowl-

[22] See, especially, the volume entitled "Education for Work," published in 1939.

edge should be taught. *Broad training in mathematics*, science, design, social subjects, and cultural subjects *is just as important in a vocational school as in any other kind of school. . . .*

Employers have imposed higher general education requirements for new employees and nearly always give preference to applicants with special training.

Youths are in difficulty today in respect to employment *unless they can meet higher standards. . . . They must have had a satisfactory general education. They must be prepared to do some kind of work well.*[23]

IV. *The Rôle of Secondary Mathematics in a Comprehensive System of General Education.* We have seen that the J C R and the P E A R look upon mathematics as a permanent ingredient of any worthwhile plan of general education. But their conception of general education is not quite the same. And a similar diversity of opinion may be observed throughout the length and breadth of our recent literature on educational objectives and school programs, and so long as the conflicting tendencies which we have just considered are agitating the minds of administrators and teachers, we may expect to find much confusion in the teaching of every major school subject.[24] It may be desirable, therefore, to suggest certain corrections in current doctrines as to the educative functions of mathematics, and to attempt the formulation of a central theme for the organization of mathematical curricula.

First, then, we must insist that at any stage the study of mathematics is NOT primarily a matter of finance, of dollars and cents, as some recent proponents of "social mathematics" seem to think. To be sure, economic training is important. But the heavy, almost exclusive emphasis on financial backgrounds around which so many curriculum plans now revolve, is not only extremely one-sided, but also most depressing to many pupils, especially to those who come from under-privileged homes.

Again, it should be obvious that mathematics is NOT merely the language of *size*, nor merely the "grammar of measurement and counting," as Professor Hogben seems to suggest. Vastly important as these arts are, they do not represent completely the essence of modern mathematics.

It can easily be shown that the two domains mentioned above by no means exhaust the potential educational significance of mathematics. At least two other major concerns of mathematics should receive their due share of attention. The first of these has to do with a task that is the common province of both mathematics and science.

For what is it that has made the modern world possible? What is back of its countless discoveries and inventions, back of our engines, our airplanes, automobiles, ocean liners, tunnels, bridges, radios, telephones, our entire industrial era?

Is it not, in the last analysis, the close coöperation of mathematics and science? Is it not the work of a Galileo, a Newton, a Leibniz, a Faraday, a Maxwell?

The power generated by this unified approach is the most dramatic story of our era. It has taught us that this world is controlled by laws. The discovery, the codification, and the application of these laws was made possible, in large measure, by *modern mathematics*. And so, to an amazing degree, it is true that

"*Mathematics is the Key and Applied Mathematics is the Tool wherewith Man Conquers the Universe.*"

Fortunately, this view of *mathematics as "the handmaiden of the sciences"* (E. T. Bell), as the foundation of technology and modern industry, as the builder and transformer of our cities, as the dynamic leaven underlying our new system of transportation, is becoming familiar to our teachers and to our educated citizens. It was given the widest publicity through John Norton's famous mural painting entitled "The Tree of Knowledge," poster copies of

[23] See "New York State Education," April, 1940.

[24] For an excellent summary of the prevalent variety of curriculum approaches, see Norton and Norton, Foundations of Curriculum Building, Ginn & Company, 1936.

which are now found in hundreds of classrooms. But it is not yet an integral part of our mathematical curricula. Dramatic support for this broader orientation will be found in such books as Whitehead's "Science and the Modern World," which should be read by every teacher of mathematics. Let it be recalled, as Whitehead says, that "mathematics supplied the background of imaginative thought with which the men of science approached the observation of nature. Galileo produced formulae, Descartes produced formulae, Huyghens produced formulae, Newton produced formulae . . . The birth of modern physics depended upon the application of the abstract idea of periodicity to a variety of concrete instances."

But even this account does not tell the whole story. If we stopped here, our total program would, in the last analysis, still be primarily utilitarian. For many centuries, however, people have also been attracted to mathematics by considerations that had little or nothing to do with monetary returns or even with social and technical progress.

Let is be recalled that both the J C R and P E A R, while doing full justice to the service values of mathematics, are even more concerned with *mathematics as a mode of thinking*, with desirable *attitudes*, *ideals*, and types of *appreciation*. That is as it should be. Is it not clear that the amazing progress of science and technology is due, after all, to speculative *thinking*, to patient experimentation, to research involving both induction and deduction? And was it not a passionate interest in truth for its own sake that led to the phenomenal growth of pure mathematics? *It is this aspect of mathematics which is in great danger at the present time.* At any cost, it must be safeguarded and restored. "For back of our industries, back of our inventions and our social institutions, there stands—as the originating and organizing force that keeps all things moving—the trained mind of the *thinker*." Pascal said, "*Our sole dignity consists in thinking.*"

Perhaps we are now ready for the following summarizing statement:

The all-embracing, common enterprise of mathematics and science is the study of an ordered universe with the aid of an ordered mind, undertaken both for its own sake and for the continuous improvement of human living.

With so broad and challenging a program before us, will it be impossible to secure for mathematics the place it so richly deserves in our schools and colleges? There is every reason for believing, with Professor Hotelling, that "*the educated man or woman of the coming generation cannot neglect to study mathematics.*"[25]

V. *Some Implications and Suggestions for Teachers of Mathematics.* Enough has been said to make it clear that teachers of mathematics are facing a very great task of reorientation. We are being attacked by both friends and foes. What shall we do about it? Certainly, the usual tinkering with curricula, such as omitting a topic here and there, moving an item back and forth, either horizontally or vertically, making the whole course easier and easier, avoiding all technicalities and obsolete areas, will not save the day for mathematics. Some of these transformations are, of course, desirable. But they are essentially surface adjustments. Instead, we must honestly face the underlying issues, correct fundamental weaknesses, and build a new, scientifically sound mathematical edifice. This will take time, And it will transcend the strength and the equipment of the individual teacher. Only a *coöperative* procedure will bring the desired results. The following three suggestions, based on extensive experience, are believed to be of vital importance in dealing with the coming reorganization.

1. *Provision for More Scientific Experi-*

[25] For a further discussion of this whole theme, see "Mathematics as a Universal and Permanent Element in Education," by William Betz, Teachers College Record, November, 1937.

mentation and for Coöperative Curriculum Planning. In facing modern educational problems, the "individual teacher approach" has definitely proved to be too unreliable and too slow. Too many factors have entered the picture to make such a "pedestrian" approach feasible. A really efficient procedure, it is now believed, demands the *coöperative* effort of subject-matter teachers, educational theorists, psychologists, administrators, sociologists, and testing experts. The *combined equipment* of such a group of investigators will be necessary in the future, to realize the desired results. That is, it has been found a wasteful procedure merely to have committees of teachers, usually after school hours, work out new curricula by a series of conferences, then to "try out" these theoretical programs in a few schools, and finally to "prescribe" them for general use. As was stated previously, of more than 3,000 mathematical courses constructed in this manner, Dr. Bruner's staff pronounced only 13% as meritorious.

This is not a sudden discovery. As early as 1926, Dr. Rugg outlined the need for a group approach in his well-known historical review of curriculum construction in American schools[26] He said, in part,

> It is clear, then, that the day has passed in which a single individual professor, teacher or administrator, psychologist, educational lawgiver or research specialist, can hope to master the manifold, highly professional tasks of curriculum-making. They are far too difficult and complex for any one person to hope to compass them all singlehanded. In this connection no generalization is of more far-reaching importance than that *the proper construction of the curriculum demands the coöperation of several specialists equipped in various fields. Curriculum-making is a coöperative enterprise.*
>
> At least five special fields of work are represented in the total enterprise of constructing a curriculum for a public-school system: (1) the study of contemporary American life—the physical and natural world, economic, political, and social institutions, culture,—every aspect; (2) the study of child capacities, interests, rates of learning, etc.; (3) educational administration—child accounting, organization of classes, curriculum materials, library facilities, the daily program and the like; (4) educational measurement, statistical methods, and controlled experimentation; (5) the professional study of specific fields of subject matter, including specialized documentation and authentication.
>
> The argument to this point shows very clearly that American education can be reconstructed out of the materials of American life only by *the coöperation of many well-equipped agencies . . .*

[26] See, especially, pp. 50–53 of the Twenty-Sixth Yearbook of the National Society for the Study of Education (1926), Part I, "Curriculum-Making, Past and Present."

This view has recently been endorsed by the Joint Committee on Curriculum, in its volume on "The Changing Curriculum" (1937, Henry Harap, Editor). In Chapter VI of that volume, Dean Melby describes the way in which the "educational forces" may be organized for more efficient service and "creative leadership." He says that "*the coöperative approach seems most feasible.*"

Nevertheless, the cornerstone of this coöperative approach will continue to be the actual *classroom experience* and work of outstanding *subject-matter teachers* and *specialists.* No amount of theory can ever replace that or be substituted for it. The so-called "expert" who has not faced the daily routine of teaching pupils, year after year, who has not himself encountered the baffling problems of pupil adjustment, of backwardness, retardation, indifference, and the like, and who is at a safe distance from the "trenches" in which teachers have to carry on the nation's constant campaign against ignorance, in order that an enlightened democracy may emerge,—such an "expert" is hardly in a position to make wise suggestions concerning more effective curricula.

How can such a coöperative approach be organized? Three phases of that plan have already been tested with considerable success.

First, in some of our larger communities, carefully chosen schools have been designated as "curriculum centers." In such "centers" definite curriculum problems are being investigated by teachers who

seem well qualified for such types of practical research. In each case the general plan to be followed is outlined in advance, at least tentatively, by a carefully chosen central advisory board. The lesson details are, however, left to the classroom teachers concerned with the new experiments. Typical lessons are observed, discussed, and criticized by one or more experienced colleagues. A record is made of each day's work, indicating the "case history" of each unit attempted. This daily report pertains to such items as pupil preparation, types of motivation, pupil participation, classroom activities, difficulties encountered, ground covered, evaluation, and the like. Such a cumulative record, carefully compiled, is considered to be the only safe point of departure for the construction of really workable curricula.

Second, at regular intervals, the teachers of the various curriculum centers meet with the advisory board for further consultation, and for a discussion of their respective experiences. Each forward step is thus given the benefit of the collective scrutiny of the group. Also, necessary corrections or revisions can at once be suggested for subsequent trial.

Third, from time to time, as the experiments progress, a larger advisory body, composed of other than subject-matter teachers and specialists, is given the opportunity to review and appraise the progress made by the active curriculum workers.

2. *The Creation of "Service Centers."* It has been urged that the individual teacher should not be expected to face the problem of curriculum revision unaided. In like manner, much help must be extended in the direction of creating a "mathematical atmosphere" in each classroom and throughout the school system. Many suggestions have been offered along this line in recent years.[27] It is gratifying that the J C R, in Appendix IV, has furnished some excellent advice on an up-to-date "equipment of the mathematical classroom," together with helpful bibliographies.

We should no longer expect a teacher of mathematics to depend exclusively on a textbook and a piece of chalk. Precisely as in a modern science classroom, there should be at hand illustrative and enrichment materials of many types, such as models, charts, slides, posters, source books and folders, instruments, designs, outlines, and even films. Notable progress has been made in this direction. An ever increasing number of classrooms is being equipped in this manner. Filing cabinets and projection lanterns are being installed, squared blackboards are being added, and even some apparatus for scientific experiments is being created.

But beyond that, there arises the imperative need of a *city-wide* attack on the same problem. What have we done to aid our schools in this way? What are we doing to interpret mathematics to the average citizen? Very little. And yet, in many communities it would not be difficult to make a beginning in this direction. Thus, a large city like New York is really one huge laboratory in mathematics. From the Battery to the Bronx we are forcibly reminded of mathematical backgrounds at almost every turn. How can we dramatize these rich relationships in such a way that they may be more widely understood and appreciated?

There seems to be no better procedure than that of arranging to establish in each community a centrally located "Hall of Mathematics." It might be installed in a museum building, in a library, or in a school that has ceased to be overcrowded.

Such a "mathematical center" should contain (1) a permanent display, constantly improved, dealing with mathematics in the modern world; (2) a growing collection of enrichment materials used to illustrate and dramatize the subject in the classroom, sent out and loaned upon re-

[27] See, for example, the Third Yearbook, the Eighth Yearbook, and the Eleventh Yearbook, of the National Council; also, numerous articles in THE MATHEMATICS TEACHER. The annual exhibits arranged by the National Council have served as a powerful stimulus.

quests from the schools; (3) a core library containing the most helpful and illuminating literature on the meaning and the significance of mathematics and its applications; (4) a curriculum laboratory for the use of committees and for group conferences.

Finally, there should be definite provision for giving to educated laymen a further insight into the purposes and the contributions of modern mathematics, through the instrumentality of public lectures, special exhibits, school dramatizations, and the like. A good beginning along these lines has already been made in a number of our large cities.

3. *Professional Growth through "Training Centers."* In the Eleventh Yearbook of the National Council of Teachers of Mathematics, Professor Wheeler was quoted as saying that "education has much of its work to do over," that we must have "an altogether different type of teacher training," and that educators must "build a new temple." And Professor Kandel has recently outlined this teacher training problem in no uncertain terms. He said, in part,

> There are signs everywhere that the traditional concept of secondary education in terms of an academic curriculum is giving place to a new concept of adolescent education—that it is the function of the school *to discover what a pupil can do and to help him to do it.*
>
> In both elementary and secondary education *the teacher assumes a new rôle.* His duty is no longer limited to imparting the dry bones of a prescribed syllabus, but he is called upon to exercise initiative, to display more enterprise, to use his imagination harnessed to common sense, and to show greater resourcefulness and versatility. He must try to understand his pupils and their environment, and to discover how the two can best be adapted to each other.
>
> This does not mean, indeed, that each teacher can go his own way; he must have guidance, and that guidance should come from a broad general education and professional preparation, from an appreciation of functions with which he is entrusted, from the suggestions and advice of his superiors, from consultation with his colleagues, and from the standards of attainment to be expected from his pupils at various stages in their development. . . .
>
> The task before the teacher is not simple; it will make greater demands upon him, but while he will enjoy a greater measure of freedom than before, he must accept the responsibility of producing men and women for the work and leisure of their world with minds well trained and characters well formed, for both are developed not in a vacuum of abstractions but in relation to the environment and culture in which they exist. *The most important change that is desirable if teaching is to become a profession is a change of emphasis in teaching from the subject and the child to the teacher, for, "as is the teacher, so is the school."*[28]

And so, is it not obvious that, no matter how many years we have been teaching, we must all "go to school" again? But where and how can this necessary training be obtained?

The writer discussed this problem at some length in a previous publication.[29] At this point, he can only refer to that analysis of a vexing situation. The teacher of tomorrow is to be a *human engineer.* No task could possibly be more inspiring than that, nor more challenging or exacting. Only unremitting self-discipline and continuous growth can make us worthy of, and fit for, such a lofty enterprise. Its difficulties are enormous, and its economic rewards are far from inviting. In more senses than one, a teacher is *born*, not made.

As yet, the typical normal school or college has done very little to prepare its graduates for the new program in education that lies ahead. Only too often, courses in "education," in methods, psychology, child development, evaluation, and the like, are conducted by specialists who have not taught in an elementary classroom for years, and know only at second hand about the pressing problems of the moment. There can be, and should be, much value even in such courses. Thus, thorough training in musical *theory* is an indispensable prerequisite for musical

[28] Kandel, I. L. *Conflicting Theories of Education*, The Macmillan Company, New York, 1938, pp. 43 ff.

[29] See the *Eleventh Yearbook of the National Council of Teachers of Mathematics* (1936), pp. 128–132.

composition. Most certainly, however, the direct application of theoretical training in the classroom is not an automatic affair. As in every other art, one learns to do only by *doing.* In like manner, much time can be saved and costly mistakes can be avoided in teaching if continuous training is provided for teachers-in-service, *administered by those who have themselves acquired the necessary experience through years of actual service in the classroom and through an unremitting program of professional growth.*

And so, in addition to "curriculum centers" and mathematical "service centers," we should have, in every community that can possibly afford it, at least one "training center" for teachers-in-service.

It is neither necessary nor possible to outline here the details of such a training program. Perhaps it is sufficient to say that it must provide for a much greater perspective, for a broader type of general culture, for a really informed professional orientation, and for a greatly increased classroom efficiency.

Much has been said in recent years about the *integration* of subject-matter fields. The proponents of such plans say little or nothing about the manner in which harassed teachers-in-service may acquire the vast increase in fundamental types of training which genuine integration would demand.

It is a commonplace that there has been an almost incredible advance in the whole range of human knowledge. In mathematics alone, according to Professor Bell, it would take no less than twenty richly endowed men a lifetime to assimilate the accumulated resources of the science. What is a busy teacher to do in the face of such a stupendous challenge?

Again, how many of us have had time for at least a glimpse of the newer psychologies of learning? What do we know about the teaching of concepts, about motivation, transfer, maturation, and the like? How many have realized that *drill* is very far from being the essence of the teaching and learning process, that Thorndike himself has dethroned mechanical repetition from its former rôle of omnipotence?

But the magnitude of the problem does not absolve us from the necessity of facing it. We must either go forward or admit defeat in the greatest crisis of human history. If we meet the future bravely, our reward is bound to be great.

"In the school of tomorrow the teacher will regain the esteem he once enjoyed, because he will be an artist, a scholar and a friend. He will direct the activities of the classroom with consummate skill, stimulating and effecting desirable mental, moral, and spiritual growth. And the community of tomorrow will reward him by granting him the freedom of his great profession, economic security, and a position of confidence and trusted leadership."

Summary and Conclusion. We have examined the present educational scene, however briefly and imperfectly, with particular reference to secondary mathematics. We have reviewed current trends and have offered critical appraisals of certain movements that are affecting the development of mathematical curricula in our American schools. How the new national reports seem to be related to this general picture, was a major concern of this discussion. Their central messages were outlined, and their divergent, though supplementary, points of view were set forth. The attempt was made to suggest a comprehensive statement as to the rôle of mathematics in the modern world. Finally, attention was given to the problem of scientific curriculum research, to modes of creating a wider appreciation of mathematics, and to a more adequate teacher-training program.

In conclusion, it should be evident that a very interesting and challenging situation is now before us. Much work will be necessary if we wish to "teach mathematics into its rightful place." But we should not become discouraged over the

magnitude of the task. For we have powerful allies in this struggle. The undeniable fact is that the world is "incurably mathematical." It would almost seem that the Great Architect who created all things is a "master mathematician," as Jeans has said.

We are living in an ordered universe over which we may have an increasing share of dominion if we approach it reverently, with the aid of ordered minds.

That is the great truth which underlies the teaching of modern mathematics and science.

It is this inspiring, world-embracing vision with which we must acquaint our citizens and our young people. With such a grand view as the basis of our new curricula, there is no reason to despair of the future of mathematics. More clearly perhaps than Plato himself we now realize that "mathematics is the science of the eternal." And that is why we may confidently believe in the lasting contribution of mathematics in the construction of the "new temple" of education that is forever in the making.

Two Important Deliberative Reports Concerned with Mathematics in the Schools

By HARL R. DOUGLASS
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IT IS rare indeed that in any year there would appear two deliberative reports of national committees in any field as challenging as "Mathematics in General Education"[1] and "The Place of Mathematics in Secondary Education."[2]

THE PROGRESSIVE EDUCATION ASSOCIATION REPORT

"Mathematics in General Education" very properly concerns itself with a review of the same answers of the teaching of mathematics to changing social conditions and psychological theories affecting secondary education and enrollments in mathematics classes. The teacher who wishes to learn just what this thing is which people are calling *general education* may be somewhat disappointed by the degree to which the report forces its readers to employ inference and implication. The statement of purpose of general education: "to provide rich and significant experiences in the major aspects of living, so directed as to promote the fullest possible realization of personal potentialities and the most effective participation in a democratic society" is not as devoid of specific meaning as a first reading would indicate, but nevertheless would seem to include all types of education for all types of pupils including specific vocational education or specific college preparatory education for specialized groups.

To the authors of the report the aims of education are apparently largely confined to the development of the personality in a general way, that is: developing personal characteristics essential to democratic living such as "social sensitivity," "esthetic appreciation," "tolerance," "cooperativeness," "self-direction," "creativeness," and the "disposition and ability to use reflective thinking in the solution of problems."

Following a brief and somewhat unsatisfying discussion of the "role of mathematics in achieving the purpose of general education," 180 pages is devoted to training pupils in problem solving and its various component steps and processes. This ably prepared treatise, constituting more than forty per cent of the report, might well appear in any volume on the teaching of mathematics, as also might the 25 pages devoted to a good discussion of "The Nature of Mathematics."

Understanding the Student consists of a condensed discussion of mental hygiene of adolescents as teachers should view it, and a case study indicating some applications in the life of "Paul," a very bright, but none too representative, problem child. The discussion of *evaluation* of student achievement is written from the modern point of view, emphasizing the measurement of growth of the *whole* child, including all personality characteristics which would be affected by experiences involved in the study of mathematics. The illustrated applications, however, are constituted largely of material calculated to measure progress along mathematical lines.

The report is noteworthy both for its modern point of view and the splendid material on problem solving, the nature of mathematics and evaluation, all of which might well appear in books not dealing especially with general education, but also for its failure to come very definitely to

[1] "Mathematics in General Education"—Final Report of a Committee of The Progressive Education Association.

[2] "The Place of Mathematics in Secondary Education"—Final Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics.

grips with the place of mathematics in *general education*. There is lacking any systematic or direct approach to the problem of re-organizing the curriculum in mathematics.

There is also lacking any discussion of what types of mathematical course of study materials would likely be better than the conventional contents of mathematics courses—no picturing of the possibilities in the way of mathematics suited to the problems of the general population for home, for health, for safety, for government, and for all the areas of life in which participation and problems are common to all. Why a more direct discussion of mathematical education for common needs was omitted may have resulted either from:

(a) the desire on the part of the committee not to offer patterns or blue prints, to keep the discussion largely a theoretical and background treatment, or

(b) the definition by the authors of "general" education as meaning in large part the development of "general" personality qualities such as "tolerance," "ability to solve problems," etc. The large amount of space given to the development of personality as well as to care given to state the objectives of education in terms of general personality outcomes leads one to believe that the latter consideration was at least very influential.

There is still a need for the development of a different type of statement relative to mathematics in general education—one which is more specific, more definitely constructive and suggestive, and one which presents the problems of mathematics instruction from the point of view of its contributions to training for specific areas and problems common to all general needs.

The Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics

The typical teacher of mathematics will find the latter of these two important publications more satisfying. It is written in the language which teachers of mathematics understand. It is more conservative in its point of view. It is more given to specific and concrete statement and illustration of what the authors have in mind.

The introduction to this pronouncement consists of a good brief statement of the role of mathematics in civilization, not essentially different from others which have appeared from time to time. The first two chapters are devoted to a few concise discussions of the relationship of education to objectives of education and emphasizes particularly the objectives to which mathematics may make outstanding contributions such as: ability to think clearly; ability to use information; ability to use fundamentals; desirable attitudes, interests and appreciations. These discussions, while seemingly parallel to the corresponding discussions in "Mathematics in General Education," are written from the point of view of developing these outcomes for their direct contributions to the objectives of education in general, and through the medium of a well rounded, integrated or healthy personality.

The discussion of the place of mathematics in education—in achieving the objectives of education—is brief and meaty —not complete or impartial.

Guiding Principles: The general discussion of the Mathematics Curriculum, Chapter IV, includes an excellent statement of what the authors think are sound "Guiding Principles" governing the selection of materials for instruction in mathematics in Grades 7–12. Some of those appealing to the author as most noteworthy are the following from pages 55–58:

> 1. (2) In contrast with this permanent foundation, we have the equally important fact that mathematics has had a remarkable growth and has been extended to widely varied fields of application.
>
> For every type of pupil, a mathematical course of study must give constant attention to the "foundations," while at the same time it stresses significant applications within the learner's potential range of understanding and interest.

2. Due attention must be given at all times not only to logical considerations, but also to psychological and pedagogical principles.

3. (4) Extensive experience has led to the conviction that in the case of retarded pupils, modifications are needed in the rate of progress and the degree of comprehension, rather than in the choice of the basic mathematical units. (See also Chapter VII.)

4. (5) The precise scope and degree of emphasis to be given to each major type of work, in a particular school, cannot be stated with finalty in any general discussion. On the contrary, these items must be regarded as subject to further local experimentation, in the light of actual time schedules and of desired or possible types of application and of training.

5. (a) Early in each year the mathematical maturity of each pupil should be determined. In case the required information is not available from reports, inventory tests may be needed to determine the amount of ground that may be covered during the semester, as well as the necessary amount of reteaching.

6. (c) An understanding of the concepts and principles of mathematics is the key to its successful study. To teach in such a way that the concepts become clear is the hardest and the most significant task confronting the teacher of mathematics. By way of illustration, a definition should usually be the outgrowth, not the beginning, of a learning process.

7. (d) "Overviews" and motivating discussions are valuable as directing guides, while summaries and organic reviews are effective means of creating perspective and confidence. A properly constructed curriculum will give adequate attention to such considerations.

8. (e) In the past, much dependence was placed on mere drill. Recent psychological investigations suggest that all techniques should be based on insight. This implies that adequate practice is to be provided, not mere drill, to lead the pupil to proper assimilation and mastery.

9. (f) Modern psychology has proved the effectiveness of "spaced learning." That is, "bunched learning" is not so productive of lasting values as "spaced learning." With slow pupils, especially, the idea of a periodic return to the same topic, providing for its growing mastery and enlarged application, is of the utmost importance. Experience shows that we cannot expect "one hundred per cent mastery" after a single, brief exposure.

10. (2) Even in a reduced program, the study should emphasize problem solving and modes of thinking, and should not become a mere sequence of formal and relatively abstract drills.

11. (3) If a unit organization is followed, it is not always advisable to attempt in each of the units a complete or exhaustive treatment of the central theme or topic under discussion. On the other hand a unit should not include unrelated "odds and ends."

12. (4) In general, a new topic should not be introduced unless there is a sufficient background of prerequisite concepts and skills to permit unhindered concentration upon the new elements.

13. (5) A new idea or principle should not, as a rule, be introduced prior to the time at which it is needed or may be effectively applied.

The Curriculum-Grade Placement: When the matter of what should be taught is under consideration, in wide variance from the report of the P.E.A. committee, the report of the joint commission is long on concrete details and short on supporting discussion. It includes, with a minimum of discussion, what it regards as a minimum list of topics to be taught or abilities to be developed and means of improving the instruction emphasizing auxiliary agencies such as films, clubs, exhibits, etc.

Two plans of organization and grade placement of instructual materials for grades seven through twelve are given along with very useful discussion of some of the more important considerations supporting the plans.

Roughly, these plans are as follows:

Plan One

7th and 8th grade—largely arithmetic, continued attention to computational skills with whole numbers, fractions, and decimals; percentage and some of its applications, intuitive and other informal geometry; graphic representation, simple formulas, with application to all fields of life.

8th—Maintenance of computational skills, extension of percentage to business and banking problems, etc., more informal geometry—angles, triangles, circles, and other figures; simplest algebraic equations; graphical representation; foundation preparation for trigonometry.

9th—Largely algebra; maintenance of arithmetical skills; informal geometry applied to algebra, beginning on trigonometry ratios.

10th—Largely plane geometry—some related algebra; maintenance of arithmetical skills; graphs; elementary trigonometry.

11th—Largely algebra and trigonometry.

12th—Solid geometry, analytic geometry, algebra, differential calculus.

PLAN TWO

7th and 8th—Similar to Plan One.

9th—Two optional plans—

(a) General mathematics; review arithmetic, graphs, much algebra, but not the more complicated manipulations; introductions to trigonometry.

(b) Algebra.

10th—Chiefly geometry; a little more algebra and trigonometry than the traditional course.

11th—Algebra.

12th—Four elective semester courses: trigonometry, solid geometry, social economics, arithmetic, and college algebra.

These two plans are more alike than many would have liked to have had them. The first is principally a description with a few modifications of the type of the traditional curriculum organization which has been increasingly regarded as inadequate to meet the mathematical needs of the high school population of the present, though constituting a good college preparation for future students of engineering, mathematics, and physics in college. It is to be commended, however, for its recognition of the need for maintaining and developing skills in arithmetical computation in which the great majority of high school graduates, who do not go on to college, as well as the small minority who do go to college, are found to be so woefully weak. It is also to be commended for its suggestions concerning correlation of different branches of mathematics.

One version of Plan Two, which provides for general mathematics in the ninth grade, is apparently based on the purpose to make secondary school mathematics adapted to the abilities and needs of a larger group than Plan One is suited for. It is also closer to the apparent needs of the mass of high school juniors and seniors in that it has provided for a course in senior civic-economic arithmetic.

Need for a Plan of Curriculum Organization for the "Other Half": Almost half of the high school students will not take mathematics beyond the eighth grade under either of these plans. That they are prepared for life with what mathematics they will remember from what they learned in elementary school is an assumption one would hardly care to defend. That pupils can best be taught the mathematics of home-making, investments, purchasing, social-security, shop, farm, and business, while they are immature elementary school children, is another unsupportable principle implied in present-day practices and in the plans offered in the Joint Commission.

Neither plan has taken up much of the slack created by the greatly increased enrollments in secondary schools. Neither plan is well suited to education of more than half of the general run of high school students. It is surprising that no plan was offered for the others. The two plans offered were for the specialist. A third plan for "general" education should have been outlined for those who do well not following the pattern of the specialist—a plan perhaps similar to those given below.

PLAN A (For large schools)

General mathematics: largely the application of arithmetic, informal geometry, and elementary algebra to farm, shop, home-consumer problems, travel, transportation, communication, health, safety, leisure, etc.; continued maintenance and development of computational skills, no arithmetic. One semester each year, grades 8 through 12, (or the equivalent). Required of all not taking other mathematics course.

In schools, in which Plan A is employed for general education, mathematics for the specialist might be organized as supplementary to the above as shown for grades 10, 11 and 12 for plan B.

PLAN B (For schools of less than 100 or 125 pupils—in grades 9 through 12—to which class more than half of the high schools belong)

Grades 7, 8, 9—Training through problem-solving in the applications of arithmetic, simple algebra, and informal geometry to all phases of life activities, as indicated in Plan A; also maintenance and development of computational skills.

In schools in which Plan B is employed in General Education, mathematics for the specialist might well be organized somewhat as follows:

Grade 10—Algebra, equivalent to second and third semester algebra.

*Grade 11—Plane and solid geometry.

*Grade 12—Algebra and trigonometry, or senior general mathematics.

*Given in alternate years.

All of these things in the report of the Commission require but 119 pages. Also in this concise report of 250 pages are several other excellent discussions of, and recommendations relative to, problems that face teachers of mathematics. Suggestions for the organization of mathematics courses for junior college constitute one chapter of the report.

Education of the Mathematics Teacher

The Commission "believes that professional training is an important part of the preparing of the teacher." The present author disagrees enthusiastically with the Commission when it states that the most important element in professional training is student practice teaching. He feels equally certain that it is of more importance that they should understand the philosophy of American education and the problems that beset American democracy and the ideals it stands for.

The Commission soundly recognizes the need for continued professional study and growth and that this does not take place automatically as the outcome of experience. The teacher should continue to grow through formal courses, through in-service training, through independent reading and study, or through some combination of these.

As would be expected, and soundly, the Commission stands unequivocally for thorough and extensive training in mathematics, and stresses the history of mathematics. Wisely, the Commission recognizes that many teachers of mathematics (in many states, the majority) teach two or more subject fields and prescribes for them a less ambitious program of training in mathematics.

Other Features

The Appendix constitutes very useful and stimulating presentations. For example, "Analysis of Mathematical Needs" includes material of a type most badly in need of study by the teacher otherwise adequately prepared to teach mathematics—the teacher who knows much of mathematics and methods, but little of the applications of mathematics to its varied uses in life.

This treatment as well as several other passages in the report furnishes encouraging evidence of the slow but steady emergence of the teaching of mathematics from a provincial, exclusive concentration upon mathematics for science and engineering, and from a lack of concern for its numerous valuable uses in business, the social sciences, in home, health, and other types of applications.

The report presents, in a few pages, a surprising amount of the best scientific information concerning the status of the transfer of training phenomenon, concluding quite correctly that, it would seem, (a) that many of the older experiments were unreliable, (b) that transfer does take place, (c) that if it is to be achieved in a satisfactory degree or amount, mathematics must be skillfully taught with generalization and transfer in mind.

The Commission puts itself on record as favoring ability grouping, but also suggests other means of caring for differences in ability and furnishes very excellent summaries of the characteristics of bright and of dull children, and further, what seems to the author, a summary of the best we know concerning adaption of instruction to bright and to dull youngsters.

The discussion of evaluation of pupil progress in mathematics is in line with modern thinking in that area and points out some of the unfortunate outcomes of the "objective test," e.g., the tendency to fail to yield valid measures of some important outcomes of the teaching of mathematics. Further the Commission suggests the need of measurement of growth of abilities in pupils along lines not strictly mathematical, that is, understanding the role of definitions and of assumptions in argument and discussions.

The Commission believes that the teacher of mathematics should be one of good character, of a personality suited to dealing with people especially youngsters, and one of powerful civic interests, and general intelligence and culture.

Summary

Both of these reports should be examined carefully by all teachers of mathematics and all educators and administrators having any responsibility for influencing the program of mathematics in the secondary schools. "Mathematics in General Education" is somewhat bulky and verbose. It should be digested and reprinted in an edition of from 75 to 100 pages. It constitutes an excellent supplement to the report of the Commission—and gives expression to some points of view little appreciated by teachers of mathematics and with which they need more contact.

The report of the Joint Commission is the most important document on this subject since the report of the National Committee on Mathematical Requirements which appeared in 1923 under the title of "The Reorganization of Mathematics in Secondary Education" and will probably be even more influential than was that bulky, none too widely read, symposium. Ably done, its greatest weakness is its conservatism. Written by persons, half of whom are chiefly interest in "the cream of the crop" who come to them for advanced training in mathematics and the other half of whom includes several known for their conservative views, the report lacks, in the discussion of some questions, notable in that relating to the curriculum, the daring that might characterize the pronouncements formulated by men equally concerned for the improvement of the teaching of mathematics but less likely to follow the grooves of tradition.

The report should be followed by a supplement upon the "Place and Program of Mathematics in the Education of the General High School Population" in which there is clearly recognized the need for guidance in the development of a program of mathematics for all high school pupils as well as for the minority who will have need of training in mathematics for advanced vocational or other specialized purposes.

Already curriculum theory and practice are going far ahead of the recommendations of these two reports. The integrationists are advocating the absorption of mathematics in a unified core-curriculum and acquiring mastery of mathematics through incidental learning. The weaknesses of that sort of thing are numerous, obvious, and insuperable. As indicated by the contents of a recent mimeographed report of a Committee scores of schools are attacking the problem of mathematics for general education individually and variously. Even if the recognized leaders in the teaching of mathematics neglect this field, it will be tilled by the rank and file and by the less well known national leaders of secondary school teachers of mathematics. They and their problem cannot be ignored. They need help and guidance. Their problem must be attacked by those who will not suffer too greatly from fear of losing caste with the traditionalists.

Does Mathematics Belong in General Secondary Education?

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There is no use trying to guess how important these two reports will be. Guessing will not affect the answer which will be given five or ten years from now. But it is not guessing to say that these reports are important, because they are written by important groups of people, and they deal with an important problem—namely, does mathematics belong in the general secondary education of the great mass of American boys and girls?

This discussion of the two reports will do the following three things: compare the audiences aimed at, compare the authors, and compare the things said.

The two books are aimed at substantially the same audience—at secondary school teachers of mathematics primarily, and also at high school principals, school superintendents, and curriculum specialists.

The authors of *The Place of Mathematics in Secondary Education* (hereafter called POMISE) are college mathematicians and teachers of mathematics in secondary schools and teachers colleges. Several of them are thorough students of curriculum problems. The Commission acknowledges the help of Professor Harl Douglass, well known for his studies of general education.

The authors of *Mathematics in General Education* (hereafter called MIGE) are college mathematicians, teachers of mathematics in secondary schools, students of general education, and students of adolescence. Professors Hartung and Bennett were very active members of the Committee and were also members of the Commission which prepared POMISE.

It is an important fact that both groups brought mathematicians and educationists together in their reports. This cooperation before publication doubtless prevented conflict after publication. Instead of the spectacle of an organization of scientific scholars violating all the canons of scientific procedure by making dogmatic and unproved assertions about their natural enemies, the educationists—a sight by no means rare—these two groups present the pleasing view of men of good will talking over their differences and coming out with constructive proposals to which they all can agree.

Both reports are good-humored, and show a sense of proportion about the importance of their subject which other groups might well imitate. Of course, they show a bias in favor of mathematics. A commission of professors of Patagonian, if asked to study the place of Patagonian in general secondary education, would certainly find that their subject had an important contribution to make. Only the professors of economics seem to feel that their subject should not be taught at the secondary school level.

In discussing the authors of the two reports it is well to mention Arnold Dresden, whose name does not appear on either list. As President of the Mathematical Association of America, Professor Dresden created the committee which later became part of the Joint Commission that wrote POMISE. He also attended several meetings of the committee that write MIGE, and took an active part, particularly in the earlier discussions. As Chairman of the Commission on Mathematics of the College Entrance Examination Board, he helped to revise the examination procedure in ways that encouraged experimentation and change in the mathematics program.

Passing on to the most important part of this comparison of the two reports, it will be useful to consider first the points of agreement.

Both reports agree that mathematics

has an important place in the general secondary education of American boys and girls. This is not surprising, but nevertheless it is important.

Both reports agree that there is no one best pattern of instruction in mathematics. Both agree that the teaching of mathematics must vary with various kinds of students, and that it must vary with changing social conditions which bring a changing youth population into the secondary schools. This was not altogether to be expected. It is possible to find a commission of mathematicians that would prescribe a particular method and content for teaching mathematics as the best for all students at all times.

Both reports stress the teaching of mathematics as a *way of thinking* so that it will be applied to the practical and theoretical affairs of life. Neither claims that mathematics taught without explict attention to generalization will result in generalized problem-solving ability. Both make the training of the mind to solve significant problems of living the primary aim of mathematics in secondary education.

Both reports emphasize evaluation of the results of mathematical instruction as a key to improvement of instruction. They describe the advances that have been made in testing in recent years, and urge the teacher to use a variety of evaluation instruments to get evidence on student achievement of a variety of outcomes hoped for from the study of mathematics.

Both reports attempt to combine the view (illustrated by Lancelot Hogben in *Mathematics for the Million*) of mathematics as a response to the practical needs of human life with the view of mathematics as a product of the human mind in its search for abstract truth. This was not entirely to be expected. Many mathematicians would reject Hogben's position.

The two reports differ, without necessarily contradicting each other, at the following points.

POMISE proposes several detailed curriculum plans and course outlines for high school and junior college mathematics. These are presented in the form of examples of the application of a set of principles fully stated for the guidance of teachers and curriculum workers. They practically require the organization of mathematics for teaching purposes in the following fields: number and computation, geometric form and space perception, graphic representation, elementary analysis (algebra and trigonometry), logical thinking, relational thinking, symbolic representation and thinking. This is certainly a change from the conventional mathematics curriculum, but it has been practiced successfully by a number of teachers. In addition, POMISE recognizes the possibility of organizing mathematics instruction around "centers of interest," or concrete problems in the lives of boys and girls, such as health, normal physical development, choice of a vocation, wise selection and use of goods and services, etc. It recommends, however, that attempts to organize instruction in this way be limited to experimental classes.

MIGE takes a definitely experimental position, urging that a mathematics curriculum be built around concrete problem situations of the sort just mentioned. Only later in the course should the concepts met in this process be classified and tagged with the labels of algebra, geometry, etc. MIGE appears to lean toward the retention of a separate mathematics course, but it urges co-operation of the mathematics teacher with teachers of other subjects, both in planning and teaching. And it is difficult to see how the mathematics teacher could follow the recommendations of MIGE without inviting teachers of other subjects into his classroom or going into the classes of other teachers. Thus MIGE leaves the door open for experimental "core courses" or integrated courses in which the mathematics teacher works with other teachers.

MIGE stresses the desirability of the mathematics teacher's studying adolescent behavior and personality development so

as to understand better the effects of his teaching on the various individuals in his classes. The chapter on "Understanding the Student" may well be the most important chapter in the report. Nevertheless, it certainly will be the least read and the least understood. Perhaps in another decade all secondary school teachers will have enough basic knowledge of human growth and development to read this chapter. Quite a few of them are seeking such knowledge through reading and advanced study.

MIGE organizes its largest section under the title "Major Understandings Growing out of Mathematical Experience" about seven concepts that are basic to problem-solving: Formulation and Solution, Data, Approximation, Function, Operation, Proof, Symbolism. Teachers should organize their teaching so as to help students develop an always more mature understanding and facility in the use of these concepts for solving problems that are important to boys and girls. But these concepts should not be taught explicitly in units devoted to approximation, function, etc.

This is the most important contribution that the report makes to the theory of mathematics instruction. But the chapters in this section are somewhat abstract and the argument is not clear at all points. If this section of the report is as important as its authors hope it is, it will become the conceptual basis for the organization of future mathematics instruction. Therefore this section should repay careful study. It certainly will require it.

Does mathematics belong in general secondary education? This is the primary question to which the two reports are directed. It is no academic question, as the statistics of enrollment in high school mathematics show. Mathematics is on trial. The two groups of expert witnesses join in answering the question, "Yes—if." The jury—the American public—will pay attention to that "if." The jury will base its verdict on a scrutiny of the actions of thousands of mathematics teachers in the secondary schools of the land. If these teachers desire a favorable verdict, they will do well to read and to act on these two reports.

A Problem in Addition

(*Don't look at bottom of page 377 until you've solved it*)

THREE MEN, asking to share a hotel room, are told by the bellboy that the rate is $30, payable in advance since they are without luggage. Each gives him a $10 bill. When the boy goes to the desk he is told the room costs only $25, and the cashier gives him five one-dollar bills in change. The bellhop, knowing the guests were willing to pay $30, keeps $2 for himself and refunds $1 to each man. Each of the three has paid out $9, which amounts to $27; and the boy has retained $2: total $29.

What has become of the other dollar?

On to the Christmas Meeting of the National Council in Baton Rouge!

December 30th, 1940-January 1st, 1941. See page 332 of the November issue for the complete programme.

The Two Recent National Reports on Mathematics in General Education

By MARY A. POTTER

President, National Council of Teachers of Mathematics, Racine, Wisconsin

IF THERE were no problems in the teaching of mathematics; if all methods of procedure had been perfectly worked out; if all desirable applications of mathematics were known; if all teachers had been chosen with the greatest skill and had become accomplished artists; if all children had been created with a uniform mental, emotional, moral, and social pattern; if the relative importance of all school subjects had been exactly determined; if the world in which mathematics is used would remain static, then there would be little need of a National Council of Teachers of Mathematics, and Reports would be undesired luxuries.

But such a Utopia has never been discovered. While we are searching for it, we band together in groups to study our problems more efficiently, and we make known our findings in Reports.

A notable contribution to the better teaching of mathematics was the epoch-making Report of 1923 which recorded and encouraged a trend away from the more formal instruction of the previous decades.

Since 1923 the world has changed with unparalleled rapidity. Industries as well as armies have become mechanized; social and economic unrest have caused both peaceful and bloody wars and revolutions; science has created new products, new industries, new theories while its handmaiden, mathematics, has lost prestige in the school curriculum; the secondary school has doubled its numbers each decade and enrolled a new type of student. It is evident that such sweeping changes in world and educational conditions called for a new survey of the problems created, and their effect on the teaching of mathematics, and a review of the various efforts that have been made to solve them.

The Joint Commission of the Mathematics Association of America and the National Council of Teachers of Mathematics, working under a grant given by the General Education Board, has published its report as the Fifteenth Yearbook of the Council under the title, "The Place of Mathematics in Secondary Education." From a somewhat different point of departure, the Commission on Secondary School Curriculum of the Progressive Education Association has issued its report called "Mathematics in General Education." A summary of each of these reports with a discussion of their varying points of view and merits is ably presented by Mr. William Betz on page 339.

Of so great importance to the teachers of mathematics are these volumes, that this issue of THE MATHEMATICS TEACHER has been devoted entirely to a detailed study of them.

How greatly these reports will influence the teaching of young people in the classrooms no longer depends upon the efforts of the Joint Commission and the Commission of the Progressive Education Association. Their work is finished. Further responsibility rests upon the teachers of mathematics for whom the studies were made. With their active co-operation, the reports will be given wide publicity, will be thoughtfully read and evaluated, will be brought to the attention of the administrative officers and curriculum builders, will be discussed by groups of teachers, and the suggestions made will be tried out in the classrooms. Perfect agreement with all of the ideas presented is neither expected nor desired. It is hoped, however, that the findings given will result in increased stimulation of thought, effort, and further experiment with the resulting increased efficiency of teaching mathematics to our boys and girls. No other ideal can replace one of this kind.

Some Suggestions to Readers of Two Recent Reports on the Mathematics Curriculum

By MAURICE L. HARTUNG
University of Chicago, Chicago, Illinois

ONE WHO makes a survey of the literature relating to mathematical education is likely to experience several different sorts of reactions. He may, for example, note that the leaders of a generation ago frequently recommended types of procedure and content which have become common only recently or are still seldom used. If he believes such recommendations are good and in line with modern views, he may develop a feeling of discouragement. The forward movement of reform seems to be a tortoise-like crawl, interrupted by periods of somnolence, by backtracking, and by detours. On the other hand, as he compares the practices of the past with those of the present, he may decide that great progress has been made. Mathematical instruction in many schools today is vastly different from what it was twenty or thirty years ago. Moreover, it seems that at the present we are in the midst of a period of relatively rapid change and reorganization. Whether or not the changes which are being made are desirable is a matter of opinion, but that they are taking place can easily be established.

Any attempt to isolate the primary forces which are bringing about a reorganization of mathematical education leads one far afield, but certain secondary forces or factors can be identified without difficulty. One such factor is the publication of the reports of committees appointed by influential organizations. The report on *The Reorganization of Mathematics in Secondary Education* issued by a committee of the Mathematical Association of America in 1923 is perhaps the outstanding example. The effect of the two recent reports under discussion in this issue of THE MATHEMATICS TEACHER can be only roughly estimated at the present time. It is probable that they, like the report mentioned above, must be circulated for at least ten or fifteen years before their real influence can be assayed.

Some indication of the change in thinking about curriculum problems which has taken place is given by that fact that both the *Place of Mathematics in Secondary Education* and *Mathematics in General Education* devote considerable space to statements of general objectives for education, a topic which the report of 1923 treated only in a brief footnote. To be sure, the Commissions which prepared the recent reports were charged with the responsibility of examining mathematical education in relation to the wider educational scene. The significant thing, however, is that the need for analyses of this sort came to be recognized, and that the teachers of mathematics on the Commissions were willing to give the problem careful study. Regardless of how satisfactory or unsatisfactory these particular formulations of general purposes for education may be, they should serve to focus the attention of other teachers upon the need for *some* formulation, and to emphasize that more than ever before the teaching of mathematics should now be considered in relation to a set of purposes broader than the specialized objectives of their own field.

It is true that the literature relating to mathematical education contains many statements of what may be called general aims, using this phrase to denote objectives which are held in common by teachers of various subjects or are school-wide in scope. The development of qualities essential for good citizenship, of the ability to think, of appreciations of various sorts, are typical examples. It is not difficult to get teachers to agree upon a list of this type, but difficulty is encountered when an effort is made to trace the connec-

tions between the immediate activities of an average classroom and these co-called ultimate objectives. The readers of these reports will do well to judge them in terms of the effectiveness of the efforts made by the Commissions to bridge this gap.

The atomization of learning experiences is perhaps inevitable in day-to-day teaching. If the fine spray of facts, skills, and specific aspects of broader concepts which is shot at students is ever to be condensed into larger drops, and eventually to flow into a powerful stream (which figuratively speaking, the ability to think should be), the teacher must consciously change the temperature in the educational climate and help dig the channels of thought. Both reports seek to bring more unity into the mathematics curriculum, but the methods they suggest are quite different. The *Joint Commission* report classifies mathematical concepts and skills primarily in terms of type of content—arithmetic, algebra, geometry, and so on. The report advocates that the student should have experiences relating to each of these categories at each grade level. If successfully worked out, this recommendation should tend to break down still further what some people have called the "water-tight compartments" of algebra, geometry, and other fields. The informed reader knows, of course, that they were never really "water-tight," and that they have been leaking into each other rather extensively in recent years. He may not favor the tendency toward "integration," believing that many values inherent in the older organization are inevitably lost. This is, however, a question of relative values, and those who favor the closer integration approved at least tacitly by the Joint Commission believe that the gains outweigh the losses.

The committee which prepared *Mathematics in General Education* chose a different set of categories or major concepts as a unifying thread. They were, of course, familiar with the fact that the National Committee of 1923 recommended that the concept of *functional relation* be made the unifying idea of the mathematics curriculum. Instead of relying upon a single major concept, however, they suggested no less than eight such concepts, that of *function* being included in the list. In a sense then, their recommendation on this issue is an extension of the one given in the earlier report. The committee believed that their list of major concepts, or others like them, would prove particularly useful in unifying the mathematical aspects of the school-wide curriculum which seems to be in the process of evolution.

The reader of these reports should note carefully that the problems faced by the two Commissions were different. The Joint Commission sought to formulate recommendations which could be safely adopted in almost any school. The Committee of the Commission on Secondary School Curriculum, however, tried first to analyze the heterogeneous experiments in curriculum modification which are under way, and then to formulate recommendations for teachers of mathematics designed to make the contributions of these teachers more effective in relation to the total curriculum. The primary forces—social, economic, psychological and philosophical—which motivate curriculum reorganization are so so strong that it is very unlikely that any report could counteract them, even if this were to be considered desirable. But reports can perhaps *direct* them. The directive influence will depend upon the adaptability of the recommendations to the immediate situation in which reorganization is to be attempted. Thus it is to be expected that one or the other of these reports will prove most helpful in a given situation. But it seems to the writer that both have a role to play, and that they supplement each other. The teacher who for one reason or another is reorganizing along the lines discussed in *Mathematics in General Education* may still find it highly profitable to examine the developing course of study in relation to that proposed in *The Place of Mathematics in Secondary*

Education. The teacher who is taking the latter as a primary guide should at least study the former in order to see the extent to which certain of its recommendations may also be incorporated.

These reports will have served one of their major purposes if they stimulate teachers of mathematics to think critically about their programs. They should also stimulate much careful educational experimentation. Although for certain sorts of questions evidence on which to base a decision is not lacking, it is surprising how many judgments about the curriculum must be based on experience and carefully considered opinion. It is to be hoped that in the future a greater number of recommendations can be based upon educational research. With the rapidly increasing interest in more refined statistical methods, such as those using careful experimental design and the analysis of variance, there is reason to believe that the scientific method as applied to education is still in its infancy. Until experiments have provided us with definite answers, we should regard many of the recommendations in both reports as tentative—that is, we should suspend judgment concerning their ultimate validity, but proceed on the basis that they represent the best judgment of Commissions which have given relatively mature thought to each question. In the meantime such experiments should be started

EDITORIALS

Are Mathematics Teachers Conservatives or Progressives?

Often when some more conservative-minded person wants to condemn the more liberal type of person he refers to him as a "progressive." Conversely, when some more liberal-minded person wishes to impugn the motives of the more conservative type he refers to him as a reactionary. However, this settles nothing, one way or the other. As Gilbert and Sullivan put it in "Iolanthe," "Every boy and every gal that's born into this world alive is either a little Liberal or else a little Conservative." But we all know that very often a person who in political affairs is pretty much of a "left-winger" is very far to the "right" in educational matters. Besides, to refer to the field of mathematical education, the realization of valid objectives which on the whole should be very much the same for the liberal and conservative groups of teachers is the main goal. Moreover, it is contrary to the facts to assume that teachers of mathematics are divided into two groups only. That is why this issue of The Mathematics Teacher is devoted to a consideration of the two recent national reports on "The Place of Mathematics in Education" of two somewhat different groups. We believe that they are different only in method, not in their fundamental desires to see mathematics occupy its rightful place among the great fields of knowledge. As a matter of fact, some persons have served on both groups. In order to get the most out of these two reports we have asked representatives of both groups to contribute to this issue and also Professor Douglass, who was on neither Commission, also to write something.

The main thing we have in mind is to get both of these reports before the mathematics teachers of the country and to other educational leaders as well, so that they can be discussed and certain conclusions reached as to the needs for the reorganization of mathematics in the schools. Obviously the reports are different, but they both serve the same general purpose, and it is to be hoped that all mathematics teachers will unite in seeing to it that these reports are given wide publicity in their local situations.

As Professor Edwin H. Reeder recently put it, "The question whether a given school is Progressive or not is not nearly so important as another question; namely, is it progressing? Is it progressing through careful anslysis of ideas and continuous experimentation with techniques toward ever higher standards in the education of children?" In such a time as this we must all be united in trying to find out the next best steps in mathematics education.

W. D. R.

The Need for More Members

It is not uncommon for a teacher of mathematics to drop his membership in the National Council of Teachers of Mathematics because, as he says, his school, his library, or his department has paid for the ensuing year. If the teacher in question reads the magazine regularly, that is a good thing, but the Council needs the support of all individual teachers. What would happen if each of our present membership took this stand and dropped his connection with the only organized group devoted primarily to the interests of mathematical education in elementary and secondary education.

In one of the largest cities in this coun-

try (and the situation is doubtless as bad in many other places), only one in six of the mathematics teachers in the schools are individual members of the Council. It will be most helpful if present members will join in a drive to get as nearly 100% membership as is possible in all schools.

The state representatives (see the list on page 335 of the November, 1940 issue) are doing a noble service in getting new members and in holding old ones in line. But the state representatives need the help of all those who are interested in keeping mathematics on a high plane. W. D. R.

New Chairman of State Representatives

Mr. A. E. Katra of the University-High School at Urbana, Ill., who has ably served for more than a year as chairman of State Representatives for the National Council of Teachers of Mathematics, has resigned and Mr. Kenneth Brown of Percy, Oklahoma has been chosen to take his place.

This work which was so well begun and carried on by Mrs. Florence Brooks Miller of Shaker Heights for some time is a most important feature of the publicity work of the Council and Mr. Katra gave a great deal of time and thought to extending the work begun by Mrs. Miller. His resignation was due mainly to his feeling that the Chairman should be more accessible to the office of The Mathematics Teacher. We hereby extend him our thanks.

Mr. Brown has started off his work with great enthusiasm and he will no doubt carry on the work most successfully.

W. D. R.

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IN OTHER PERIODICALS

By NATHAN LAZAR
The Bronx High School of Science, New York City

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2. Reports of the Tests and Measurements Committee," pp. 3–7.
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6. Swinden, B. A., "The Half Angle Formulae and the Tangent Rule," pp. 118–119.
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2. Kennedy, E. C., "Root Isolation Through Curve Analysis," pp. 373–378.
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2. Brill, Elmer, "Ode to an Exponential Curve," p. 80.
3. Keyser, Cassius Jackson, "The Role of Mathematics in the Tragedy of our Modern Culture," pp. 81–87.
4. Nordgaard, Martin A., "The Pedagogic Ideas of Paul La Cour," pp. 88–94.
5. Davidson, Gustav, "The Most Tragic Story in the Annals of Mathematics—The Life of Evariste Galois," pp. 95–100.
6. Watson, E. C., "College Life in the Days of Stokes, Cayley, Adams, and Kelvin," pp. 101–106.
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8. "Memorabilia Mathematica," pp. 118–119.
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There Wasn't Any Other Dollar

THE BRAIN TWISTER on page 369 is deceptively stated. The $27 paid by the three men *included* the $2 kept by the dishonest bellboy. So his $2 should be subtracted from the $27, not added to it. That gives you $25 kept by the cashier, plus $2 kept by the bellboy, plus $3 refund: $30.

The Frontispiece for This Issue

THE frontispiece for this issue was contributed by Miss Josephine Wagner, South Dakota Representative at Sioux Falls.—EDITOR.

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